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M. A./M. Sc. (First Semester)

EXAMINATION, 2024-25

MATHEMATICS

Paper Second

(Abstract Algebra–I)

(Math–002)

Time : Two hours] **[Maximum Marks : 60**

Note : *Attempt any* ***four*** *questions. All question carry equal marks.*

1\. (a) Let a be any element of a group b than prove that two elements $x, y \in b$ give rise to the same conjugate of a if and only if they give belong to the same right caset of the normalizer of a in b.

(b) Let b be a finite group and z be the centre of b. Then show that the class equation of b can be written as :

(P.T.O.)

$$o\ (b) = \frac{\sum o(b)}{afzo\,[N(a)]} + o(z)$$

When the summation runs over the element a in each conjugate class containing more than one element.

2.(a) Let Z be the centre of a group b. Show that if $a \in z$, then the cyclic sub-group $\{a\}$ of b generated by a is a normal subgroup of b.

(b) Show that the emy quotient group of a cyclic group is cyclic and the converse need not be time.

3.(a) Show that any homomorphic image of an abelian group is abelian and the converse need not be true.

(b) Suppose b is a group and N is a normal subgroup of b. Let f be a mapping from b to $b\ /\ N$ defined by

$$f(x) = Nx\ \forall\ x \in b.$$

Then show that f is a homomorphism of b onto $b\ /\ N$ and kernal $f\ = N$.

4. Show that the set $I(b)$ of all inner automorphisms of a group b is a normal subgroup of the group of its automorphisms isomorphic to the quotient group b/z of b where z is the centre of b.

5. (a) Define composition series of a group and give an example.

(b) If b is a group and N is a normal subgroup of b such that both N and b / N are solvable, prove that b is solvable.

6. (a) If H, K are two subgroups of a group b such that $b = H \times K$, then show that H, K are normal subgroup of b, and $b / H \cong K$. and $b / K \cong H$.

(b) Prove that every abelian group of order 6 is cyclic.

7. (a) Show that S is an ideal of $S + T$ where S is an ideal of ring R and T any subring of R.

(b) Prove that a commutative ring R with identity is a field if and only if it has no proper ideals.

8. (a) If R is an arbitry ring and R' is the set of constant polynomials in $R[x]$, then show that R' is isomorphic to R.

(b) Prove that a polynomial domain $F[x]$ over a field F is a principal ideal ringh.