234121

S-250 (N)

B.A. (Fourth Semester)
NEP EXAMINATION 2023-24
MATHEMATICS
(Abstract Algebra)
[SOS/Maths./C-064]

Time : Two Hours] *[Maximum Marks : 70*

Note:(i) Attempt any five questions from Section A and any three questions from Section B.

(ii) Answer each question of Section A within 50 words.

(iii) Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

Section-A

Note: Attempt any five questions. Each question carries five marks.

1. If G is a group and a, b, c $\in$ G. Then prove that $ab=ac \Rightarrow b=c$

2. Prove that cube roots of unity forms a group under multiplication.

3. Prove that if every element of a group G, is its own inverse, then G is abelian.

4. If H_1 and H_2 are two subgroups of a group G, then prove that $H_1 \cap H_2$ is also a subgroup of G.

5. Find the generators of the cyclic group:

   $G=\{a^1, a^2, a^3, a^4, a^5, a^6, a^7, a^8=e\}$

6. Prove that true alternating group An is a normal subgroup of symmetric group Pn.

7. Define a field with an example.

Section-B

Attempt any three questions:

8. (a) If a and b are any two elements of a group G, then prove that $(ab)^{-1} = b^{-1}.a^{-1}$.

   (b) The order of an element 'a' of a group is the same as that of its inverse a^{-1}.

9. (a) If f is an isomorphism from a group G to group G^1, then prove that:

   $f(e)=e^1$ where e and e^1 are identity elements in G and G^1 respectively.

(b) If H is a subset of a group G such that $ab^{-1} \in H \ \forall \ a,b \in H$, then prove that H is a subgroup of G.

10. (a) If H is any subgroup of a group G and $h \in H$, then prove that $Hh = H = hH$.

(b) Prove that every subgroup of a cyclic group is cyclic.

11. Let H and K be finite subgroups of a group G, then prove that $O(HK) = \frac{O(H).O(K)}{O(H \cap K)}$

12. (a) Explain the group of symmetries of isosceles non-equilateral triangle.

(b) If G is a group and H is a subgroup of index 2 in G. Prove that H is a normal subgroup of G.

13. Define the following with an example.

(i) Ring without unity.

(ii) Ideal

(iii) Integral domain.