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S-1137

M. A./M. Sc. (First Semester) EXAMINATION, 2023-24

MATHEMATICS

Paper Third

(Mechanics)

(MATH–003)

Time : Two Hours] [*Maximum Marks : 60*

Note : Attempt any *four* questions. All questions carry equal marks.

1. (a) Distinguish finite and impulsive forces with examples. If a rigid body be moving under the action finite forces the sum of whose moments about a given line is zero throughout the motion, the angular momentum of the body about that line remains unaltered throughout the motion.

   (b) An elliptic lamina is rotating about its centre on a smooth horizontal table. If ω_1, ω_2, ω_3 be its angular velocities when the extremity of its major axis, its focus and the extremity of its minor axis respectively become fixed, prove that $\frac{7}{\omega_1}=\frac{6}{\omega_2}+\frac{5}{\omega_3}$.

2. (a) Define conservative force and state the condition under which a force F (t, r, v) is conservative. The kinetic energy of a rigid body, moving in any manner is at any instant equal to the kinetic energy of the whole mass, supposed to be collected at its centre of inertia and moving with it together, with the kinetic energy of the whole mass relative to its centre of inertia.

(b) A uniform rod, of length '$2a$' is placed with one end in contact with a smooth horizontal table and is then allowed to fall. If a be its initial inclination to the vertical, show that its angular velocity when it is inclined at an angle θ is $\sqrt{\frac{6g}{a}\cdot\frac{\cos\alpha-\cos\theta}{1+3\sin^2\theta}}$. Find also the reaction of the table.

3. Define holonomic and degree of freedom of a dynamical system. Derive Lagrange's equations of motion and deduce it it terms of Lagrangian.

4. A uniform rod, of length '2a', which has one end attached to a fixed point by a light inextensible string of length $\frac{5a}{12}$, is performing small oscillations in a vertical plane about its position of equilibrium. Find its position at any time, and show that the period of its principal oscillations are $2\pi\sqrt{\frac{5a}{3g}}$ *and* $\pi\sqrt{\frac{a}{3g}}$.

5. (a) Derive Euler's equations of motions for a dynamical system and hence prove that angular momentum is constant when body moves under no finite forces.

   (b) A body moves under no forces in such a way that the resolved parts of its angular velocity about one of the principal axes at the centre of grativity is constant. Show that the angular velocity of the body must be constant.

6. (a) Define instantaneous axis of rotation and instantaneous axis of rotation. Prove that locus of the invariable line is a cone with vertex origin.

   (b) A body moves under no forces about a point 'O', the principle moments of inertia at 'O' being '3A', '5A' and '6A'. Initially angular velocity of the body has the components $\omega_1 = \eta$, $\omega_2 = 0$, $\omega_3 = n$ about the principal axes. Show that at any time t, $\omega_2 = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right)$ and ultimate body rotates about the mean axis.

(a) Derive Hamilton's equations of motion for a holonomic dynamical system.

(b) Write the Hamiltonian function and equation of motion for a simple pendulum.

. (a) Define Hamilton's variational principle and deduce the Lagrange's equations of motions.

(b) Use Hamilton's principle to find the equations of motion of one dimensional harmonic oscillator.