Roll No. ....

231161

S–1135

M. A./M. Sc. (First Semester) EXAMINATION, 2023-24

MATHEMATICS

Paper First

(Discrete Structures)

(MATH—001)

Time : Two Hours] [*Maximum Marks : 60*

Note : Attempt any *four* questions. All questions carry equal marks.

1. (a) Let B = {1, 5, 7, 35} be the set of positive integers and operations + and . are defined as follows :

$$a+b=\text{lcm}(a,b), a.b=\text{gcd}(a,b)\ \forall a,b\in B$$

Show that (B, +, ., ') is a Boolean algebra.

P. T. O.

(b) In the Boolean algebra B, express the Boolean function :

$$f(x,y,z) = (x+y).(x+z').y+z'$$

in its disjunctive normal form.

2. Write short notes on the following :

(i) Directed and Undirected Graph

(ii) Simple Graph

(iii) Degree of a vertex

(iv) Complete Graph

(v) Subgraph

3. (a) Solve the recurrence relation :

$$a_{n+2} - 5a_{n+1} + 6a_n = 2$$

with initial condition $a_0 = 1$ and $a_1 = -1$.

(b) Use generating function to solve the recurrence relation :

$$a_n = 3a_{n-1} + 2;\ a_0 = 1$$

4. (a) Define a relation R on the set Z of all integers as follows :

$$mRn \leftrightarrow m+n \text{ is even } \forall m,n \in Z.$$

Is R a partial order relation ? Prove or give a counter examples.

(b) Define poset and draw the Hasse diagram for the poset :

$$\left[P(\{a,b,c\}),\subseteq\right]$$

5. (a) Explain Karnaugh's map for three variables and use it to simplify the Boolean expression :

$$X = A'B'C' + A'BC' + ABC' + AB'C$$

(b) Prove that the sum of degree of the vertices in an undirected graph is even.

6. If L be any lattice, then for any $a,b,c \in L$ show that :

(i) $a \vee a = a, \quad a \wedge a = a$

(ii) $a \vee b = b \vee a, \quad a \wedge b = b \wedge a$

(iii) $a \vee (b \vee c) = (a \vee b) \vee c, \quad a \wedge (b \wedge c) = (a \vee b) \wedge c$

(iv) $a \vee (a \wedge b) = a, \quad a \wedge (a \vee b) = a$

7. (a) Show that the maximum number of edges in a simple graph with n-vertices if $\dfrac{n(n-1)}{2}$.

(b) Define linear homogeneous and non-homogeneous recurrence relation and solve :

$$a_n = a_{n-1} + 2a_{n-2} \quad n \geq 2;$$

$$a_0 = 0, \; a_1 = 1$$

8. (a) Let $A = \{1, 2, 3, 4\}$ and consider the relation $R = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 3), (3, 4), (4, 4)\}$. Show that R is a partial ordering and draw its Hasse diagram.

(b) State and prove Handshaking theorem of the graph theory.