Roll No. ....

233162

S–3240

M. A./M. Sc. (Third Semester)

EXAMINATION, 2023-24

MATHEMATICS

(Differential Equations)

(MATH—C—014)

[Ti]me : Two Hours] [*Maximum Marks : 60*

[No]te : Attempt any *four* questions. All questions carry equal marks.

(a) Using the Picard's iteration method, find the third approximation of the solution of the equation

$$\frac{dy}{dx} = z, \ \frac{dz}{dx} = x^3(y + z).$$

where $y = 1, z = \frac{1}{2}$ at $x = 0$.

(b) State Picard's theorem on existence of solutions of differential equations. Show that

$f(x, y) = xy^2$ satisfies the Lipchitz condition on the rectangle $R :|x| \le 1, |y| \le 1$ but does not satisfy a Lipschitz condition on the strip $S :|x| \le 1, |y| < \infty$.

2. State and prove Sturm separation theorem. Consider the equation $y'' + y = 0$ and the solutions $\phi_1(x) = \cos x$, $\phi_2(x) = \sin x$. Verify sturm separation theorem.

3. Using Frobenious method, solve the Legendre's differential equation :

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0, \ n \in R.$$

4. Define the ordinary and singular points. Discuss the singularities of the differential equation :

$$x^2y'' + xy' + (x^2 - n^2)y = 0 \text{ at } x = 0 \text{ and } x = \infty.$$

(a) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to Canonical form and solve it.

(b) Classify the partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 u}{\partial z \partial y} = 0.$$

6. Using the method of separation of variables, to solve the equation :

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$$

Given that $v = 0$ when $t \to \infty$ as well as $v = 0$ at $x = 0$ and $x = l$.

7. Solve the wave equation :

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\left(\frac{\partial^2 y}{\partial t^2}\right), \text{ for } 0 \le x \le 2\pi,\ t \ge 0$$

and $y(x, 0) = \sin^3 x;\ 0 \le x \le 2\pi.$

8. An infinitely long uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. The end is maintained at 100°C at all points and the other edges are at 0°C. Find the steady state temperature function $u(x, y)$.