Roll No. ....

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M. A./M. Sc. (Third Semester) EXAMINATION, 2023-24

MATHEMATICS

Paper XVII

(Calculus of Variations)

Time : Two Hours] [*Maximum Marks : 60*

Note : Attempt any *four* questions. Each question carries 15 marks.

1. (a) Show that when a function f does not depend on x explicity, that is $f = f(x, y, y')$, then :

$$f - y'\frac{\partial f}{\partial y'} = c \text{ (constant)}$$

(b) Find the extremal of the functional :

$$\mathrm{I}[y(x)] = \int_a^b \left(y^2 + y'^2 + 2ye^x\right)dx$$

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2. (a) Show that functional :

$$\int_0^1 \left[2x + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]$$

such that :

$$x(0)=0,\ y(0)=0, x(1)=1.5,\ y(1)=1$$

is stationary for :

$$x = 1 + \frac{t^2}{2},\ y = t.$$

(b) Determine the extremal of the functional :

$$I[y(x)] = \int_{-l}^{l}\left(\frac{1}{2}\mu y''^2 + \lambda y\right)dx$$

subject to :

$$y(-l)=0,\ y'(-l)=0,\ y(l)=0,\ y'(l)=0.$$

3. (a) Find the extremal of the functional :

$$I = \int_0^{\pi}\left(y'^2 - y^2\right)dx$$

under the conditions $y(0)=0,\ y(\pi)=1$ and subject to the constraint :

$$\int_0^{\pi} y\,dx = 1.$$

(b) Find the plane curve of fixed perimeter and maximum area.

4. (a) Find the shortest distance between parabola $y^2 = 4x$ and straight line $x + y = -5$.

(b) State and prove principle of least action.

5. (a) State and prove Hamilton-Jacobi's equations.

(b) Discuss the Jacobi and Legendre conditions for extremum for the functional :

$$I[y(x)] = \int_0^1 \left(\frac{x^2 y'^2}{2} - 2xyy' + y \right) dx$$

$$u(0) = 0, \text{ where } u = \delta y.$$

Further, derive the extremal satisfying $u(1) = \frac{1}{2}$ and emanating from (0, 1).

6. (a) Show that the transformation :

$$Q = p + iaq, \quad P = \frac{p - iaq}{2ia}$$

is canonical.

(b) Prove that the functional :

$$I[y(x)] = \int_1^2 \frac{x^3}{y'^2} dx$$

satisfying the condition :

$$y(1) = 1, \quad y(2) = 4$$

has weak extremum 3/8.

7. Solve the boundary value problem :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=2x^2;$$

with boundary conditions $y(0)=, y(1)=1$ by using Galerkin's method.

8. Solve the boundary value problem :

$$y''+y=e^x \text{ and } y\left(\frac{\pi}{2}\right)=0$$

by collocation methods.