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# 233161
# S–3239

## M. A./M. Sc. (Third Semester)
## EXAMINATION, 2023-24

MATHEMATICS

(Topology)

(MATH—C—013)

*Time : Two Hours* ] [ *Maximum Marks : 60*

**Note :** Attempt any *four* questions. All questions carry equal marks.

1. (a) Let T be the collection of subsets of N consisting of empty set $\phi$ and all subsets of the form

$$Gm = \{m, m+1, m+2, .....\}, m \in N$$

show that T is a topology for N. What are open sets containing 5 ?

(b) Show that the intersection of two topologies for X is again a topology for X

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2. (a) Prove that in a topological space, the neighbourhoods of a point satisfy the four axioms of neighbourhoods.

(b) Let (X, T) be a topological space and let A, B be any subsets of X. Then show that :

(i) $X^\circ = X, \phi^\circ = \phi$

(ii) $A^\circ \subset A$

(iii) $A \subset B \rightarrow A^\circ \subset B^\circ$

(iv) $(A \cap B)^\circ = A^\circ \cap B^\circ$

(v) $A^\circ \cup B^\circ \subset (A \cap B)^\circ$

(vi) $A^\circ = A$

3. (a) Prove that a mapping $f$ from a space X into another space Y is continuous if and only if :

$$f(\overline{A}) \subset \overline{f(A)} \text{ for every } A \subset X.$$

(b) Let $f$ be a mapping of R into R defined by :

$$f(x) = \begin{cases} x & \text{when } x \leq 1 \\ x + 2 & \text{when } x > 0 \end{cases}$$

Show that $f$ is not U – U continuous.

(a) Show that two disjoint sets A and B are separated in a topological space (X, T) if they are both open and closed in the subspace $A \cup B$.

(b) Show that if (X, T) is disconnected and T′ is finer than T, then (X, T′) is disconnected.

5. (a) Let Y be sub-space of X. Then show that Y is compact if and only if every covering of Y be sets open in X contains a finite sub-collection covering Y.

(b) Prove that a subset A of R is compact if and only if A is bounded and closed.

6. (a) Let (X, T) be a first axiom space. Then show that if there exists a monotone decreasing local base at every point of X.

(b) Prove that every second countable space is first countable.

7. Give an example to show that a $T_1$-space need not be $T_2$.

8. Define the following with example :

(a) Base for a topology

(b) Limit point

(c) Connectedness

(d) First countable space