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M. A./M. Sc. (Third Semester) EXAMINATION, 2023-24

MATHEMATICS

(Differential Geometry)

(MATH—E—001)

Time : Two Hours] [*Maximum Marks : 60*

Note : Attempt any *four* questions. All questions carry equal marks.

1. (a) Find the osculating plane at the point on the helix :

$$x = a\cos u, y = a\sin u, z = cu.$$

(b) Prove that a necessary and sufficient condition that a curve plane, is that :

$$[r', r'', r'''] = 0.$$

2. (a) For the curve $x = 3t, y = 3t^2, z = 2t^3$, prove that $\rho = -\sigma = \frac{3}{2}(1 + 2t^2)^2$.

P. T. O.

(b) Find the equation of the osculating sphere at the point (1, 2, 3) on the curve :

$$r = (2t + 1, 3t^2 + 2, 4t^3 + 3).$$

3. (a) If O, P are the adjacent points of a curve and the arc OP = s, show that the difference between the chord OP and the arc OP is :

$$\frac{s^3}{24\rho^2},$$

where the powers of s higher than the three being neglected.

(b) Find the equation to the developable surface which has the curve :

$$x = 6t, y = 3t^2, z = 2t^3$$

for its edge of regression.

4. (a) By considering the value of $rt - s^2$, prove that the surface $xyz = a^3$ is not developable.

(b) Find the envelope of the family of planes :

$$3a^2x - 3ay + z = a^3.$$

5. (a) Define first fundamental form and give its geometrical interpretation.

(b) Prove the formulae :

$$H\hat{N} \times \hat{N}_1 = Mr_1 - Lr_2 \text{ and}$$

$$H\hat{N} \times \hat{N}_2 = Nr_1 - Mr_2 \text{ where } \hat{N} \text{ is}$$

unit surface normal and L, M, N are second fundamental coefficients.

6. (a) Find the direction coefficients making an angle $\frac{\pi}{2}$ with the direction coefficients (l, m).

(b) Show that the curves :

$$du^2 - (u^2 + c^2)\,dv^2 = 0$$

form an orthogonal system on the right helicoid

$$r = (u \cos v, u \sin v, cv).$$

7. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature, is that :

$$\mathrm{R}\,dr + d\hat{\mathrm{N}} = 0 \text{ at each of}$$

its points, where R is the normal curvature, and $\hat{\mathrm{N}}$ is unit surface normal vector.

8. (a) Show that the parametric curves of a surface of revolution which is given by :

$$r = (u \cos v, u \sin v, f(u)), \text{ are}$$

its lines of curvature.

(b) Find the Gaussian curvature at a point of the surface :

$$x = a(u + v),\ y = b(u - v),\ z = uv.$$