Roll No. .....

233462

S–3242

M. A./M. Sc. (Third Semester) EXAMINATION, 2023-24

MATHEMATICS

(Mathematical Statistics)

(MATH—E—002)

Time : Two Hours] [*Maximum Marks : 60*

Note : Attempt any *four* questions. All questions carry equal marks.

1. (a) Define the probability. For any *two* events A and B. Show that :

$$P(AB) \leq P(A) \leq P(A + B) \leq P(A) + P(B).$$

(b) A and B throw alternatively a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. Find their respective chances of winnings. If A begins.

2. (a) State and prove additive law of probability.

(b) What is the chance that a leap year selected at random will have 53 Sundays ?

P. T. O.

3. (a) What is multiple and partial correlation coefficient ? Explain it.

(b) What is line of regression ? Derive the equation of line of regression.

4. (a) Find the first four moments of the binomial distribution.

(b) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random :

(i) 1

(ii) 0

(iii) at most 2, bolts will be defective

5. (a) Prove that Person's coefficient of correlation r lies between -1 and $+1$.

(b) The regression lines of y on x and of x on y are respectively $y = ax + b$ and $x = cy + d$. Show that the means are $\bar{x} = (bx + d)(1 - ac)$ and $\bar{y} = (ad + b) \cdot (1 - ac)$ and correlation coefficient between x and y is $\sqrt{ac}$. Also show that the ratio of the standard derivation of y and x is $\sqrt{\frac{a}{c}}$.

6. (a) If X_1 and X_2 be two dependent random variables with Poisson distributions with parameters m_1 and m_2 respectively, then the sum $X_1 + X_2$ is a random variable with Poisson distribution with parameter $m_1 + m_2$.

(b) Fit a Poisson distribution of the set of observations :

x	f
0	122
1	60
2	15
3	2
4	1

7. (a) The diameter of an electric cable is assumed to be continuous random variable with probability density function :

$$f(x) = 6x(1 - x) : 0 \leq x \leq 1$$

(i) Verify that above is a p.d.f.

(ii) Find the mean and variance

(b) Explain any *two* of the following :

(i) Normal distribution

(ii) Mode of the Poisson's distribution

(iii) Multiplicative law of probability

(iv) Tchebycheff's inequality

8. (a) Establish the formula :

$$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 = 2\pi\sigma_x\sigma_y.$$

(b) If x and y are uncorrelated random variables. Find the coefficient of correlation between $x + y$ and $x - y$.